



GCE A LEVEL – **NEW**

1305U60-1



**THURSDAY, 6 JUNE 2019 – AFTERNOON**

**FURTHER MATHEMATICS – A2 unit 6**  
**FURTHER MECHANICS B**

1 hour 45 minutes

### ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

### INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Answers without working may not gain full credit.

Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

### INFORMATION FOR CANDIDATES

The maximum mark for this paper is 80.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**Reminder:** Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A large aeroplane, of mass 360 tonnes, starts from rest at the beginning of a straight horizontal runway. The aeroplane produces a constant thrust of 980 kN and experiences a variable resistance to motion of magnitude  $(80 + 0.1v^2)$  kN, where  $v$  ms<sup>-1</sup> is the speed of the aeroplane after it has travelled  $x$  metres.

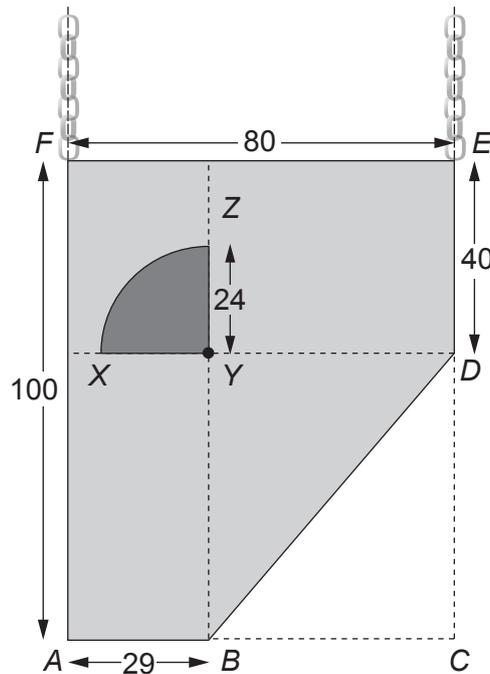
- (a) (i) Find the maximum speed that the aeroplane can attain.  
(ii) Show that  $v$  satisfies the differential equation

$$3600v \frac{dv}{dx} = 9000 - v^2. \quad [5]$$

- (b) Find an expression for  $v^2$  in terms of  $x$ . [7]
- (c) Given that the aeroplane must achieve a speed of at least 85 ms<sup>-1</sup> to take off, determine the minimum length of the runway. [2]
- (d) Explain why, according to this model, the aeroplane will not reach the speed found in (a)(i). [1]

2. A metal sign is formed by **removing** triangle  $BCD$  from a rectangular lamina  $ACEF$  made of uniform material, and **adding** a quarter circle  $XYZ$ , made of the same uniform material, with a particle attached to its vertex at  $Y$ . The sign is supported by two light chains fixed at  $E$  and  $F$ .

The quarter circle has radius 24 cm and the particle at  $Y$  has a mass equal to half of that of the removed triangle.  $XD$  is parallel to  $AC$  and  $BZ$  is parallel to  $AF$ . The dimensions, in cm, are as shown in the diagram below.



- (a) Calculate the distance of the centre of mass of the sign from
- $AF$ ,
  - $AC$ .
- [11]
- (b) The support at  $F$  comes loose so that the sign is freely suspended at  $E$  by one chain alone. Given that it then hangs in equilibrium, calculate the angle that  $EF$  makes with the vertical.
- [3]

**TURN OVER**

3. A light elastic string, of natural length  $l$  m and modulus of elasticity 14 N, is hanging vertically with its upper end fixed and a particle of mass  $m$  kg attached to the lower end. The particle is initially in equilibrium and air resistance is to be neglected.

- (a) Find, in terms of  $m$ ,  $g$  and  $l$ , the extension,  $e$ , of the string when the particle is in equilibrium. [2]

The particle is pulled vertically downwards a further distance from its equilibrium position and released. In its subsequent motion, the string remains taut. Let  $x$  m denote the extension of the string from the equilibrium position at time  $t$  s.

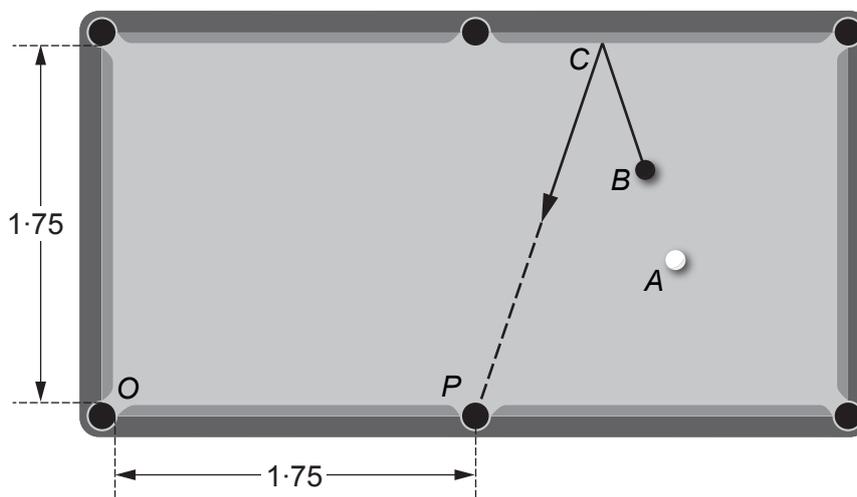
- (b) (i) Write down, in terms of  $x$ ,  $m$ ,  $g$  and  $l$ , an expression for the tension in the string.  
(ii) Hence, show that the particle is moving with Simple Harmonic Motion which satisfies the differential equation,

$$\frac{d^2x}{dt^2} = -\frac{14}{ml}x.$$

- (iii) State the maximum distance that the particle could be pulled vertically downwards from its equilibrium position and still move with Simple Harmonic Motion. Give a reason for your answer. [6]
- (c) Given that  $m = 0.5$ ,  $l = 0.7$  and that the particle is pulled to the position where  $x = 0.2$  before being released,
- (i) find the maximum speed of the particle,  
(ii) determine the time taken for the particle to reach  $x = 0.15$  for the first time. [6]

4. Ryan is playing a game of snooker. The horizontal table is modelled as the horizontal  $x$ - $y$  plane with the point  $O$  as the origin and unit vectors parallel to the  $x$ -axis and the  $y$ -axis denoted by  $\mathbf{i}$  and  $\mathbf{j}$  respectively. All balls on the table have a common mass  $m$  kg. The table and the four sides, called cushions, are modelled as smooth surfaces.

The dimensions of the table, in metres, are as shown in the diagram.



Initially, all balls are stationary. Ryan strikes ball  $A$  so that it collides with ball  $B$ . Before the collision,  $A$  has velocity  $(-\mathbf{i} + 8\mathbf{j}) \text{ ms}^{-1}$  and, after the collision, it has velocity  $(2\mathbf{i} + \mathbf{j}) \text{ ms}^{-1}$ .

- (a) Show that the velocity of ball  $B$  after the collision is  $(-3\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$ . [3]

After the collision with ball  $A$ , ball  $B$  hits the cushion at point  $C$  before rebounding and moving towards the pocket at  $P$ . The cushion is parallel to the vector  $\mathbf{i}$  and the coefficient of restitution between the cushion and ball  $B$  is  $\frac{5}{7}$ .

- (b) Calculate the velocity of ball  $B$  after impact with the cushion. [3]

- (c) Find, in terms of  $m$ , the magnitude of the impulse exerted on ball  $B$  by the cushion at  $C$ , stating your units clearly. [3]

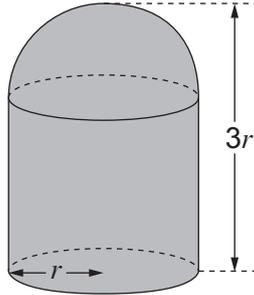
- (d) Given that  $C$  has position vector  $(x\mathbf{i} + 1.75\mathbf{j}) \text{ m}$ ,

- (i) determine the time taken between the ball hitting the cushion at  $C$  and entering the pocket at  $P$ ,  
 (ii) find the value of  $x$ . [4]

- (e) Describe one way in which the model used could be refined. Explain how your refinement would affect your answer to (d)(i). [2]

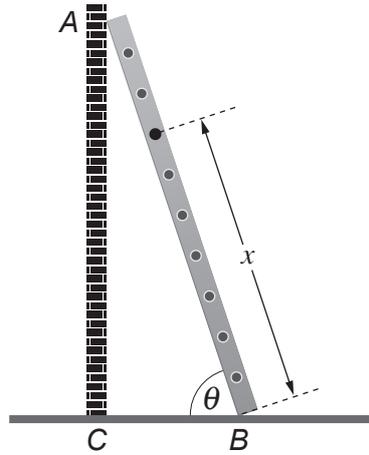
**TURN OVER**

5. (a) Show, by integration, that the centre of mass of a uniform solid hemisphere of radius  $r$  is at a distance of  $\frac{3r}{8}$  from the plane face. [4]
- (b) The diagram shows a composite solid body which consists of a uniform right circular cylinder capped by a uniform hemisphere. The total height of the solid is  $3r$  cm, where  $r$  represents the common radius of the hemisphere and the cylinder.



Given that the density of the hemisphere is 50% more than that of the cylinder, find the distance of the centre of mass of the solid from its base along the axis of symmetry. [6]

6.



A uniform ladder  $AB$ , of mass  $10\text{ kg}$  and length  $5\text{ m}$ , rests with one end  $A$  against a smooth vertical wall and the other end  $B$  on rough horizontal ground. The ladder is inclined at an angle  $\theta$  to the horizontal. A woman of mass  $75\text{ kg}$  stands on the ladder so that her weight acts at a distance  $x\text{ m}$  from  $B$ .

- (a) Show that the frictional force,  $F_N$ , between the ladder and the horizontal ground is given by

$$F = 5g \cot \theta (1 + 3x). \quad [5]$$

For safety reasons, it is recommended that  $\theta$  is chosen such that the ratio  $CB : CA$  is  $1 : 4$ .

- (b) Determine the least value of the coefficient of friction such that the ladder will not slip however high the woman climbs. [6]
- (c) State one modelling assumption that you have made in your solution. [1]

**END OF PAPER**